Unilateral Carbon Policy with Intermediates

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Abstract

I consider a two-country general equilibrium model of climate change and trade where one country (Home) imposes a carbon policy while the other (Foreign) does not. My model extends Kortum and Weisbach (2023) to incorporate intermediate goods. Intermediate goods are empirically relevant, because they account for more than 60% of the value of manufacturing output, and also highlight the distinction between direct and embodied emissions, a central concept in policy discussions. The optimal policy in my setting adds two new features: (i) a tax based on carbon embodied in intermediate goods used to produce imports and (ii) a good-specific export subsidy for *all* exported goods. The optimal policy exploits international trade and shifts the composition of intermediates used by Foreign in order to reduce embodied emissions worldwide. I show that these findings broadly support the European Union's latest carbon policy, the Carbon Border Adjustment Mechanism.

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1 Introduction

A plethora of country and region specific carbon policies has now emerged (EU ETS, China CCER etc). A growing concern with such unilateral policies is carbon leakage: firms faced with high carbon prices shift production to avoid them. There has been both theoretical arguments and empirical evidence of carbon leakage[1]. In light of these concerns, it is crucial to study how a country or a group of countries can optimally and unilaterally reduce world emissions.

Kortum and Weisbach (2021, henceforth KW) utilize a two-country general equilibrium model of trade to offer intuitive analytical solutions to the problem. One country (Home) imposes a carbon policy while the other (Foreign) does not. I extend their model to incorporate an intermediate good while retaining its analytical tractability. By introducing an intermediate good, I make the model more empirically relevant. Intermediate goods account for more than 60% of the value of manufacturing output and carbon flow datasets include them in order to accurately attribute emissions to final users. The extended model also highlights the distinction between direct and embodied emissions, a concept central to policy discussions.

My paper makes quantitative, theoretical, and policy contributions. On the quantitative front, I show that introducing intermediate goods is important because I can better calibrate the model to carbon flows.

On the theoretical front, I solve the general equilibrium model to derive new analytical solutions for the optimal policy. One of the major obstacles with modelling intermediate goods in climate trade models is the need to calculate embodied emissions. Intermediate goods introduce a recursive structure where producing a given good implicitly requires the good itself. With international trade, it becomes intractable to explicitly attribute the origins each input material. I give analytical solutions to this seemingly intractable problem in the two-country setup of KW. Two new features of the optimal policy are: (i) border adjustments on energy embedded in the intermediate good used in production and (ii) export subsidies on the intensive margin for goods with strong Home comparative advantage. The more expansive export subsidies serve to alter (make cleaner) the composition of the intermediate good in Foreign, which reduces embedded emissions in Foreign production for itself.

On the policy front, the model offers insights into the EU's new Carbon Border Adjustment Mechanism (CBAM). CBAM makes the novel proposal to impose border adjustments on all emissions embedded in EU imports. The model broadly supports the EU's proposal: (i) the optimal policy includes border adjustments on both direct and embedded emissions in imported goods and (ii) the optimal border adjustment on embedded emissions is reduced proportional to the share of intermediate goods that originated from Home. I also argue that trade regulations should make exceptions for climate related export subsidies. Although ex-

[1] Böhringer et al 2012, Aichele and Felbermayr 2015, Misch and Wingender 2021

port subsidies may harm manufacturers in a country, dirty production harms everyone in the world.

1.1 Prior Literature

This paper builds on the current trade and environment literature. Markusen (1975), Copeland and Taylor (1994), Copeland (1996) use a two-country general equilibrium model to analyze unilateral carbon policies. They indicate that the optimal tariffs should tax imports based on carbon content. Hoel (1996) derives a similar result in a partial equilibrium analysis with multiple countries. KW combines Markusen (1975) with the trade model of Dornbusch, Fisher and Samuelson (1977). They introduce energy as an input in production. In addition, they propose that the optimal policy should use a combination of Pigouvian tax on extraction and border adjustments so that producers and consumers at Home face the same energy price. Barresi (2022) and Farrokhi and Lashkaripour (2021) study the optimal unilateral policy in the context of multiple countries and industries. Both papers analyze the optimal policy with climate clubs (Nordhaus 2015) and model a game where countries can be coerced to joining the club.

I bridge the gap between trade theory and environmental economics by introducing intermediate goods. Trade theory has long modelled intermediates (Eaton and Kortum 2002) while climate change models shied away from them. One reason is the potential of intractability. Models in trade often consider the notion of value, as opposed to raw quantities, to overcome this obstacle. However, climate trade models require explicit quantities, such as energy consumption, to attribute carbon emissions to their final users. With trade and idiosyncratic production intensities across countries, analytical solutions of embodied energy are difficult to derive. This paper shows that in the two-country setup, with the same set of production assumptions as in KW, an analytical solution is possible. I characterize the optimal policy with help from tools in trade such as Leontief inverse and from value added production as used by Noguera and Johnson (2015).

The paper proceeds as follows. Section 2 describes the model setup. Section 3 solves the competitive equilibrium and points out improvements in the data calibration of KW. Section 4 solves the planner's problem for the optimal policy. Section 5 proposes a simple implementation that achieves the optimal allocation. Section 6 is self-contained and qualitatively relates the model to CBAM. Section 7 concludes.

2 Model setup

Suppose there are two countries, Home and Foreign. A planner at Home controls production, imports and exports to maximize utility, subject to constraints, while Foreign remains passive with no carbon policies.

Home (Foreign) are endowed with labour L (L^*) and energy deposits E (E^*). Labour is perfectly mobile within a given country and can be used to provide services. Suppose for simplicity that there is only one source of energy which can be extracted and costlessly traded in the international market. Manufacturers use energy which generates emissions. The disutility from emissions motivates the planner's policies.

I model the production process with a composite intermediate good. Following the setup in Dornbusch, Fisher and Samuelson (1977), goods are on a continuum indexed by $j \in [0,1]$. They are produced by combining labour, energy and the composite intermediate good. The composite intermediate good is the aggregation of all goods j; producing a unit of good j implicitly requires all goods on the continuum. Each good can be traded internationally with some iceberg cost.

2.1 Preferences

Suppose that the utility for a representative Home consumer is additively separable in utility from consuming services, C_s, composite good, C_g, and harms from climate change φQ_e^W:

$$U = C_s + u(C_g) - \varphi Q_e^W,$$

where

$$u(c) = \eta^{1/\sigma} \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma}.$$

σ is the elasticity of substitution across goods and η governs the demand between goods and services. Let Foreign have similar preferences with $u^*(\eta^*, \sigma^*)$ and φ^*.

2.2 Technology

Energy (fossil fuels) are deposited in a continuum of fields, ordered by difficulty of extraction. Let $Q_e = E(a)$, $Q_e^* = E^*(a^*)$ be the amount of energy extracted with unit labour requirement below a, a^* in Home and Foreign. World extraction is defined as $Q_e^W = Q_e + Q_e^*$.

There is a continuum of goods $j \in [0,1]$ that is produced. Producers combine labour, energy and the composite intermediate good according to the CES production function F, scaled by the total input requirement a_j (or a_j^* for Foreign), to produce

$$q_j = \frac{1}{a_j} F(L_j, E_j, N_j) = \frac{1}{a_j} L_j f(k_j, h_j)$$

units of good j. $k_j = E_j / L_j$ is the energy intensity while $h_j = N_j / L_j$ is the intermediate good intensity. Given some energy and intermediate good intensities k, h, the unit labour, energy, and intermediate good requirements are:

$$l_j(k,h) = a_j/f(k,h); \quad e_j(k,h) = kl_j(k,h); \quad n_j(k,h) = hl_j(k,h).$$

Production intensities can vary across goods and across source and destination pairs. For simplicity, denote:

$$l_j^i = l_j(k_j^i, h_j^i),$$

for $i \in \{y, x, m, y^*\}$, and similarly for e_j and n_j. Superscripts y, x are Home production for itself and for Foreign, respectively. Superscripts m, y^* are Foreign production for Home and for Foreign, respectively.

Take services, produced one to one with labour, to be numerarire. Define $\tau \geq 1$ to be the iceberg cost of exporting from Home and $\tau^* \geq 1$ from Foreign. Order goods by Home comparative advantage so that

$$A(j) = \frac{a_j^*}{a_j}$$

is decreasing. Further assume that $A(j)$ is continuous and strictly decreasing, with $A(0) = \infty$ and $A(1) = 0$.

2.3 Composite good

I introduce a composite good to parsimoniously model intermediates in production. The composite good is formed domestically according to a Dixit-Stiglitz aggregator, with elasticity of substitution of σ (or σ^*). It can be consumed as a final product or used in production, but is not traded.

Let y_j, x_j be quantities of Home production for itself and Foreign, respectively, and let y_j^*, m_j be quantities of Foreign production for itself and Home, respectively. The total amount of the composite good available in Home and Foreign is

$$Z_g = \left(\int (y_j + m_j)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \quad Z_g^* = \left(\int (y_j^* + x_j)^{(\sigma^*-1)/\sigma^*} \right)^{\sigma^*/(\sigma^*-1)}.$$

Recall that the intermediate good requirement is n_j^i. Thus, the quantity of the composite good required to sustain the level of production in Home and Foreign is

$$N_g = \int n_j^y y_j + \tau n_j^x x_j dj, \quad N_g^* = \int n_j^{y*} y_j^* + \tau^* n_j^m m_j dj.$$

Home and Foreign consume the composite good in quantities C_g and C_g^*, and use N_g and N_g^* in production. To clear markets, the amount available is equal to the amount used or consumed:

$$Z_g = N_g + C_g, \quad Z_g^* = N_g^* + C_g^*.$$

With the definition of Z_g and Z_g^*, it follows that[2]:

$$u(C_g) = \int_0^1 u((C_g/Z_g)(y_j + m_j))dj, \quad u^*(C_g^*) = \int_0^1 u((C_g^*/Z_g^*)(y_j^* + x_j))dj,$$

which implies that within each region, a fixed share of each good j is consumed while the rest are used in production as the intermediate good. The marginal utilities of consuming good j in Home and Foreign are therefore[3]:

$$u'(C_g)Z_g^{1/\sigma}(y_j + m_j)^{-1/\sigma}, \quad u^{*\prime}(C_g^*)(Z_g^*)^{1/\sigma^*}(y_j^* + x_j)^{-1/\sigma^*}.$$

2.4 Cost of inputs

Let $\mathbf{p} = [p_e, p_N]$ be a vector of energy and intermediate good valuations. They can either be the planner's value of energy and the intermediate good or market prices. Producers choose the cost minimizing production intensity vector $\mathbf{x}(\mathbf{p}) = [k(\mathbf{p}), h(\mathbf{p})]$ according to:

$$\begin{aligned}
\mathbf{x}(\mathbf{p}) &= \arg\min_{\mathbf{x}}(l_j(\mathbf{x}) + p_e e_j(\mathbf{x}) + p_N n_j(\mathbf{x})) \\
&= \arg\min_{\mathbf{x}} \frac{1 + \mathbf{p}\cdot\mathbf{x}}{f(\mathbf{x})}.
\end{aligned}$$

[2]Note that

$$\begin{aligned}
u(C_g) = u((C_g/Z_g)Z_g) &= \frac{\eta^{1/\sigma}}{1-1/\sigma}\left(((C_g/Z_g)Z_g)^{1-1/\sigma} - 1\right) \\
&= \frac{\eta^{1/\sigma}}{1-1/\sigma}\left((C_g/Z_g)^{1-1/\sigma}\int_0^1 [(y_j + m_j)]^{(\sigma-1)/\sigma}dj - 1\right) \\
&= \int_0^1 \frac{\eta^{1/\sigma}}{1-1/\sigma}\left([(C_g/Z_g)(y_j + m_j)]^{(\sigma-1)/\sigma} - 1\right)dj \\
&= \int_0^1 u((C_g/Z_g)(y_j + m_j))dj
\end{aligned}$$

The steps are identical for Foreign.

[3]From the definition of $u(x)$, we know that $u'(c) = \left(\frac{\eta}{c}\right)^{1/\sigma}$. Hence the marginal utility of consuming good j is

$$u'((C_g/Z_g)(y_j + m_j)) = \left(\frac{\eta}{(C_g/Z_g)(y_j + m_j)}\right)^{1/\sigma} = u'(C_g)Z_g^{1/\sigma}(y_j + m_j)^{-1/\sigma}$$

The cost of a bundle of inputs with the given energy and intermediate good prices is then:

$$g(\mathbf{p}) = \frac{1 + \mathbf{p} \cdot \mathbf{x}(\mathbf{p})}{f(\mathbf{x}(\mathbf{p}))}.$$

This equation shows that for any fixed $\mathbf{p}$, the production intensities are the same across all goods produced in a given country (Home or Foreign)[4]. The only variation across goods is the input requirement a_j or a_j^*.

2.5 Foreign's problem

Since Foreign is passive, I solve for Foreign's competitive outcomes. Foreign can produce good j for itself at price p_j^{y*} or import it from Home at price p_j^x. Its marginal utility of consumption and quantity of consumption are determined by

$$u^{*\prime}(c_j^*) = p_j^* \implies c_j^* = \eta^*(p_j^*)^{-\sigma^*},$$

where

$$p_j^* = \min\{p_j^{y*}, p_j^x\}.$$

Recall that a fixed share of every good j is consumed, $c_j^* = (C_g^*/Z_g^*)(y_j^* + x_j)$. Foreign's consumption can then be expressed as[5]:

$$C_g^* = \left(\int (c_j^*)^{(\sigma^*-1)/\sigma^*} dj \right)^{\sigma^*/(\sigma^*-1)}.$$

The price index of the composite good is then:

$$p_N^* = \left(\int (p_j^*)^{1-\sigma^*} dj \right)^{1/(1-\sigma^*)}.$$

[4]This result is a consequence of the production function F is common to all goods in all regions.

[5]Note that

$$\begin{aligned}
c_j^* &= (C_g^*/Z_g^*)(y_j^* + x_j) \\
(c_j^*)^{(\sigma^*-1)/\sigma^*} &= ((C_g^*/Z_g^*)(y_j^* + x_j))^{(\sigma^*-1)/\sigma^*} \\
\left(\int (c_j^*)^{(\sigma^*-1)/\sigma^*} dj \right)^{(\sigma^*-1)/\sigma^*} &= \left(\int ((C_g^*/Z_g^*)(y_j^* + x_j))^{(\sigma^*-1)/\sigma^*} dj \right)^{(\sigma^*-1)/\sigma^*} \\
\left(\int (c_j^*)^{(\sigma^*-1)/\sigma^*} dj \right)^{(\sigma^*-1)/\sigma^*} &= (C_g^*/Z_g^*)Z_g^* = C_g^*
\end{aligned}$$

This price also applies to the composite consumption good in Foreign so:

$$u^{*\prime}(C_g^*) = p_N^*.$$

Since Foreign producers can purchase the composite good at price p_N^* and energy at price p_e, the cost to produce a unit of good j there is:

$$p_j^{y*} = a_j^* g(p_e, p_N^*).$$

3 Competitive equilibrium

I analyze the effect of including the composite intermediate good in a competitive equilibrium. All goods in KW are final goods and are consumed, whereas some goods in my model are used in production as intermediates. The addition of the intermediate good highlights the distinction between energy used to produce for consumption and total embodied energy. Energy used to produce for consumption is the amount of energy used directly in production while total embodied energy also includes energy embedded in the composite intermediate good used in production. KW uses the two notions of energy interchangeably and calibrates energy used to produce final products to data on total energy embodied in final demand. I show that the calibration can be improved.

3.1 Set up

To get analytically transparent expressions for intermediate goods, suppose that the production function is Cobb-Douglas in intermediate good use, with share α_n:

$$q_j = \frac{1}{a_j} F(E_j, L_j)^{1-\alpha_n} N_j^{\alpha_n}.$$

Let p_N, p_N^* clear the composite good markets in Home and Foreign. Further, suppose p_e clears the world energy market. The cost of inputs in Home and Foreign is:

$$g(p_e, p_N) = g(p_e)^{1-\alpha_n} p_N^{\alpha_n}, \quad g(p_e, p_N^*) = g(p_e)^{1-\alpha_n} (p_N^*)^{\alpha_n},$$

where $g(p_e)$ is the input cost function for labour and energy. If $\alpha_n = 0$, then the expression collapses to $g(p_e, p_N) = g(p_e)$ as in KW.

The price of good j at Home is:

$$p_j = \min\{a_j g(p_e, p_N), \tau^* a_j^* g(p_e, p_N^*)\},$$

which is the minimum price between Home producing for itself and Home importing from

Foreign, accounting for the iceberg trade cost. Similarly, the price faced by Foreign consumers is:

$$p_j^* = \min\{a_j^* g(p_e, p_N^*), \tau a_j g(p_e, p_N)\}.$$

Consumers choose quantities of good j according to:

$$u'(c_j) = p_j \implies c_j = \eta p_j^{-\sigma},$$

$$u^{*\prime}(c_j^*) = p_j^* \implies c_j^* = \eta^* (p_j^*)^{-\sigma^*}.$$

Quantities consumed are determined by the marginal utilities of consumption and the cost of production.

3.2 Energy consumption

Denote $y_{cj}, x_{cj}, m_{cj}, y_{cj}^*$ as quantities of good j consumed in a given origin-destination pair. In this analysis with intermediate goods, we need to distinguish between energy used directly to produce for consumption for Home (Foreign), $C_{e,c}$ ($C_{e,c}^*$) , and total embodied energy, $C_{e,t}$ ($C_{e,t}^*$). In KW, with no intermediates, the two notions collapse down to the same thing. I show that with the intermediate good, $C_{e,c}$ is strictly less than $C_{e,t}$.

Energy used to produce for consumption, by origin and destination, is:

$$\begin{bmatrix} C_{e,c}^y \\ C_{e,c}^m \end{bmatrix} = \begin{bmatrix} \int e_j^y y_{cj} dj \\ \int \tau^* e_j^m m_{cj} dj \end{bmatrix}, \quad \begin{bmatrix} C_{e,c}^x \\ C_{e,c}^{y*} \end{bmatrix} = \begin{bmatrix} \int \tau e_j^x x_{cj} dj \\ \int e_j^{y*} y_{cj}^* dj \end{bmatrix}.$$

Specifically, $C_{e,c}^y$ represents the amount of energy used by Home to produce final products that are consumed at Home. Aggregating by destination, the energy used directly to produce for consumption for Home and Foreign is:

$$C_{e,c} = C_{e,c}^y + C_{e,c}^m, \quad C_{e,c}^* = C_{e,c}^{y*} + C_{e,c}^x$$

Appendix A.1 shows that the energy used by Home to produce for Home (Foreign), $C_{e,c}^y$ ($C_{e,c}^x$), as a share of the total energy consumed by Home (Foreign), $C_{e,c}$ ($C_{e,c}^*$), is equal to the import (export) threshold that characterizes comparative advantage, specifically:

$$\bar{j}_m = \frac{C_{e,c}^y}{C_{e,c}}, \quad \bar{j}_x = \frac{C_{e,c}^x}{C_{e,c}^*}. \tag{1}$$

Let G_e, G_e^* be total energy used in production at Home and Foreign, respectively. By the Cobb-Douglas assumption, a share α_n of G_e is used to produce the intermediate good. Since the intermediate good has the same composition as the composite good, equation 1 shows that $\bar{j}_m$ share of the energy used on the intermediate good at Home, $\alpha_n G_e$, can be attributed

Table 1: Energy embodied in final demand in competitive equilibrium

$C^y_{e,t} = M_{BAU}\left(C^y_{e,c} + \frac{\alpha_n \bar{j}_x}{1-\alpha_n(1-\bar{j}_x)} C^m_{e,c}\right)$	$C^m_{e,t} = M^*_{BAU}\left(\frac{\alpha_n(1-\bar{j}_m)}{1-\alpha_n \bar{j}_m} C^y_{e,c} + C^m_{e,c}\right)$
$C^x_{e,t} = M_{BAU}\left(\frac{\alpha_n \bar{j}_x}{1-\alpha_n(1-\bar{j}_x)} C^{y*}_{e,c} + C^x_{e,c}\right)$	$C^{y*}_{e,t} = M^*_{BAU}\left(C^{y*}_{e,c} + \frac{\alpha_n(1-\bar{j}_m)}{1-\alpha_n \bar{j}_m} C^x_{e,c}\right)$

This table shows the gross energy consumption for each destination-source pair. For example, $C^y_{e,t}$ is the gross amount of energy used to produce for Home consumption. They are computed by taking the Leontief inverse of equation 2.

to Home. Thus, $\alpha_n \bar{j}_m G_e$ is the amount of energy from Home that is used to produce the intermediate good. The energy market clearing condition is:

$$\begin{bmatrix} G_e \\ G^*_e \end{bmatrix} = \begin{bmatrix} \alpha_n \bar{j}_m & \alpha_n \bar{j}_x \\ \alpha_n(1-\bar{j}_m) & \alpha_n(1-\bar{j}_x) \end{bmatrix} \begin{bmatrix} G_e \\ G^*_e \end{bmatrix} + \begin{bmatrix} C^y_{e,c} \\ C^m_{e,c} \end{bmatrix} + \begin{bmatrix} C^x_{e,c} \\ C^{y*}_{e,c} \end{bmatrix}. \quad (2)$$

The total energy used in production is the sum of energy used to produce the intermediate good and energy used to produce final products for consumption. I decompose it term by term for Home. Home produces final products for itself ($C^y_{e,c}$) and for Foreign ($C^x_{e,c}$). In addition, Home produces goods aggregate to form the intermediate good used by Home and Foreign. $\alpha_n \bar{j}_x G_e$ ($\alpha_n \bar{j}_x G^*_e$) amount of energy is used by Home to produce the intermediate good in Home (Foreign). The intuition for $\alpha_n \bar{j}_x G^*_e$ is that $\alpha_n G^*_e$ amount of energy is used to produce the intermediate good in Foreign, of which $\bar{j}_x$ share comes from Home. Using the Leontief inverse, I construct the total embodied energy in Table 1 by origin and destination:

$$G_e = C^y_{e,t} + C^x_{e,t}, \quad G^*_e = C^{y*}_{e,t} + C^m_{e,t},$$

where $C^i_{e,t}$ is the total embodied energy for a given origin-destination pair $i \in \{y, m, x, y^*\}$. For example, $C^y_{e,t}$ is the total amount of energy embodied in Home's final demand for production for itself. $C^y_{e,t}$ is a linear combination of $C^y_{e,c}$ and $C^m_{e,c}$, as opposed to only $C^y_{e,c}$, because Home uses the intermediate good in producing for itself, which embodies imported goods.

Note that

$$M_{BAU} = \left(1 - \alpha_n \bar{j}_m - \frac{(\alpha_n \bar{j}_x)(\alpha_n(1-\bar{j}_m))}{1-\alpha_n(1-\bar{j}_x)}\right)^{-1}$$
$$M^*_{BAU} = \left(1 - \alpha_n(1-\bar{j}_x) - \frac{(\alpha_n \bar{j}_x)(\alpha_n(1-\bar{j}_m))}{1-\alpha_n \bar{j}_m}\right)^{-1},$$

are the intermediate good multipliers as in Johnson and Noguera (2015), applied to energy use as opposed to output. M_{BAU}, $M^*_{BAU} \geq 1$ describe the gross amount of energy needed in either Home or Foreign to supply a unit of net output for domestic consumption. So, total

Table 2: Total energy in terms of energy used for consumption

$C^y_{e,t} = D\left(1-\alpha_n\left(1-\frac{\bar{j}_x}{\bar{j}_m}\right)\right)C^y_{e,c}$	$C^m_{e,t} = DC^m_{e,c}$
$C^x_{e,t} = DC^x_{e,c}$	$C^{y*}_{e,t} = D\left(1-\alpha_n\left(1-\frac{1-\bar{j}_m}{1-\bar{j}_x}\right)\right)C^{y*}_{e,c}$

This table lists the gross energy required to produce for consumption, expressed in terms of energy used to produce for consumption. I can express them in this simple way due to the nature of the model. D is the determinant of the Leontief inverse that differs by a constant from M_{BAU} and M^*_{BAU}.

energy embodied in Home's final demand from Home production for itself is the gross amount of energy used to supply Home consumption plus the gross amount of energy embodied in Home imports that originated from Home. As a feature of the model, I express the total amount of energy embodied in final demand in terms of energy used to produce for consumption. Appendix A.2 shows an example and Table 2 summarizes the results.

To give some intuition, suppose a consumer in the US buys a car manufactured domestically. With international trade, some components of the car (steel) are imported from China, while raw material for the Chinese steel (iron) are exported from the US to China. Then it follows that the energy embodied in the car ($C^y_{e,t}$) is comprised of gross energy embodied in Home assembling the car ($M_{BAU}C^y_{e,c}$) and gross energy needed to produce the steel in China ($M_{BAU}\frac{\alpha_n\bar{j}_x}{1-\alpha_n(1-\bar{j}_x)}C^m_{e,c}$)[6].

3.3 Share of total energy embodied in final demand

Now consider energy embodied in final demand in a given source-destination as a share of energy embodied in a given destination, because it is used to calibrate the export/import thresholds in KW. In a model with no intermediate goods, energy used for consumption is equal to energy embodied in final demand. KW exploits this fact and uses data on energy embodied in final demand for calibration. I show that, under mild assumptions ($\bar{j}_x < \bar{j}_m$), it overestimates $\bar{j}_x$ and underestimates $\bar{j}_m$. The export threshold is strictly lower than the share of energy embodied in traded goods for both countries, while the import threshold is strictly higher.

[6]The definition of M_{BAU} is used to follow the notation in Johnson and Noguera (2015). An alternate way to define M_{BAU} is to normalize it by the fraction in front of $C^m_{e,c}$. It then becomes the intermediate good multiplier that describes the gross amount of energy needed in Foreign to supply a unit of net output for Home consumption.

Using results from Table 2, the shares of energy embodied in final demand are[7]:

$$\frac{C^x_{e,t}}{C^{y*}_{e,t} + C^x_{e,t}} = \bar{j}_x \sum_{k=0}^{\infty} (\alpha_n(\bar{j}_m - \bar{j}_x))^k, \quad \frac{C^m_{e,t}}{C^y_{e,t} + C^m_{e,t}} = (1 - \bar{j}_m) \sum_{k=0}^{\infty} (\alpha_n(\bar{j}_m - \bar{j}_x))^k.$$

KW calibrates the above shares to $\bar{j}_x$ and $1 - \bar{j}_m$. But for $\bar{j}_m > \bar{j}_x$, this is not true. Their calibration is only accurate when there is no iceberg trade cost (and hence $\bar{j}_x = \bar{j}_m$).

The infinite sum has a similar interpretation to the Leontief inverse. To produce a unit of good j to export, Home uses the composite good, which is comprised of goods produced at Home up to $\bar{j}_m$. As a result, Foreign implicitly consumes goods $\bar{j}_m > j > \bar{j}_x$ through importing from Home, even though it does not explictly import them. The summation of k from 0 to infinity represents the last kth stage of production. In the final stage of production, $\bar{j}_x$ share of energy used to produce that good came from Home. However, there is an additional $\alpha_n(\bar{j}_m - \bar{j}_x)$ share of energy embodied in the intermediate good used in production that originated from Home. This continues for all stages of production which makes up the infinite sum.

This example motivates the need to include intermediate goods in production. It illustrates the inherent difference between energy used to produce for consumption and energy embodied in final demand. One may have suspected that they are off by a constant factor regardless of source and destination, but in fact,

$$C^i_{e,t}/C^i_{e,c} \neq K, i \in \{y, x, m, y^*\},$$

for any constant K. Due to trade, the ratios of gross to net energy consumption are higher for traded goods ($C^x_{e,t}/C^x_{e,c} > C^y_{e,t}/C^y_{e,c}$). This emerges due to home bias. Foreign implicitly consumes goods it does not import because they are embodied in the intermeidate good used by Home.

[7] Substituting from Table 2 and using equation 1:

$$\begin{aligned}\frac{C^x_{e,t}}{C^{y*}_{e,t} + C^x_{e,t}} &= \frac{C^x_{e,c}}{C^x_{e,c} + \left(1 - \alpha_n\left(1 - \frac{1-\bar{j}_m}{1-\bar{j}_x}\right)\right) C^{y*}_{e,c}} \\ &= \frac{\bar{j}_x}{1 - \alpha_n(\bar{j}_m - \bar{j}_x)} \\ &= \bar{j}_x \sum_{k=0}^{\infty} (\alpha_n(\bar{j}_m - \bar{j}_x))^k.\end{aligned}$$

4 Planning problem

Now consider the problem with an optimal policy at Home. I solve the model to find the optimal border adjustment on imports and exports, while noting key insights that are brought by introducing the intermediate good in production. The outer problem remains largely unchanged, but the inner problem offers insights to import and export thresholds under the optimal policy.

4.1 Planner's Lagrangian

The planner allocates resources at Home to maximize welfare subject of labour, energy, composite good and Foreign welfare constraints. As in KW, I substitute labour, and Foreign welfare constraints. I additionally add Lagrange multipliers for goods market clearing conditions. As a result, the objective is equivalent to the planner maximizing world utility, subject to energy and goods constraints:

$$\begin{aligned}
\mathcal{L} = {} & u(C_g) + u^*(C_g^*) - \varphi^W Q_e^W - L_e^W \\
& - \int l_j^y y_j + \tau l_j^x x_j + l_j^{y*} y_j^* + \tau^* l_j^m m_j dj \\
& - \lambda_e \left(\int e_j^y y_j + \tau e_j^x x_j + e_j^{y*} y_j^* + \tau^* e_j^m m_j dj - Q_e^W \right) \\
& - \lambda_N \left(\int n_j^y y_j + \tau n_j^x x_j dj + C_g - Z_g \right) \\
& - \lambda_N^* \left(\int n_j^{y*} y_j^* + \tau^* n_j^m m_j dj + C_g^* - Z_g^* \right).
\end{aligned}$$

The planner dictates production intensities of all goods that are produced or consumed at Home. Differentiating with respect to k_j^y and h_j^y, we notice that $l_j^y + \lambda_e e_j^y + \lambda_N n_j^y$ enter the objective as $a_j g(\lambda_e, \lambda_N)$, similarly for:

$$\tau^*(l_j^m + \lambda_N e_j^m + \lambda_N^* n_j^m) = \tau^* a_j^* g(\lambda_e, \lambda_N^*), \quad \tau(l_j^x + \lambda_e e_j^x + \lambda_N n_j^x) = \tau a_j g(\lambda_e, \lambda_N).$$

Hence, the Lagrangian can be rewritten as:

$$\begin{aligned}
\mathcal{L} = {} & u(C_g) + u^*(C_g^*) - \varphi^W Q_e^W - L_e^W + \lambda_e Q_e^W \\
& - \int a_j g(\lambda_e, \lambda_N) y_j + \tau a_j g(\lambda_e, \lambda_N) x_j dj \\
& - \int (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) y_j^* + \tau^* a_j^* g(\lambda_e, \lambda_N^*) m_j dj \\
& - \lambda_N (C_g - Z_g) - \lambda_N^* (C_g^* - Z_g^*).
\end{aligned}$$

4.2 Goods market at Home

I first optimize over C_g at Home. This yields a characterization of the planner's shadow price of the composite good:

$$\frac{\partial \mathcal{L}}{\partial C_g} = u'(C_g) - \lambda_N \implies \lambda_N = u'(C_g).$$

The planner's shadow price is equal to the marginal utility of consumption, which shows that it does not regulate the composite good's price at Home. This seems intuitive but one can argue that the planner *should* regulate it because it embodies goods produced in Foreign which is produced using a dirtier energy intensity. The argument for not regulating the composite good's price at Home is that the planner can add policies upstream so that components of the intermediate good are produced cleanly. As I will show in the remainder of the section, this is precisely the case.

4.3 Inner problem

Now consider the inner problem where I optimize over each good j by source and destination, except for Foreign production for itself. Foreign's outcomes are determined competitively in Section 2.5. Similar to KW, the inner problem is

$$\begin{aligned}\mathcal{L}_j = &-a_j g(\lambda_e, \lambda_N) y_j - \tau a_j g(\lambda_e, \lambda_N) x_j - \tau^* a_j^* g(\lambda_e, \lambda_N^*) m_j - (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) y_j^* \\ &+ \lambda_N \left(\int (y_j + m_j)^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)} + \lambda_N^* \left(\int (y_j^* + x_j)^{(\sigma^*-1)/\sigma^*} \right)^{\sigma^*/(\sigma^*-1)},\end{aligned}$$

where the first line is the input costs for producing good j for an origin-destination pair. The second line is planner's shadow values of production in Home and Foreign.

4.3.1 Goods for consumers at Home

The first order condition for y_j is

$$-a_j g(\lambda_e, \lambda_N) + \lambda_N Z_g^{1/\sigma} (y_j + m_j)^{-1/\sigma} \leq 0,$$

with equality if $y_j > 0$. First order condition for m_j is

$$-\tau^* a_j^* g(\lambda_e, \lambda_N^*) + \lambda_N Z_g^{1/\sigma} (y_j + m_j)^{-1/\sigma} \leq 0,$$

with equality if $m_j > 0$. Equating the two, I characterize the threshold good $\bar{j}_m$ such that both conditions hold with equality:

$$A(\bar{j}_m) = \frac{g(\lambda_e, \lambda_N)}{\tau^* g(\lambda_e, \lambda_N^*)}.$$

This good separates Home production for itself from imports. Home produces goods $j < \bar{j}_m$ while Foreign produces goods $j > \bar{j}_m$. Contrary to KW, where $A(\bar{j}_m) = 1/\tau^*$, this threshold is no longer invariant under policy. It instead depends on the planner's values for Home and Foreign's composite good.

Concretely, suppose that production is Cobb-Douglas in the intermediate good with share α_n. $\bar{j}_m$ is characterized by:

$$A(\bar{j}_m) = \frac{\lambda_N^{\alpha_n}}{\tau^* (\lambda_N^*)^{\alpha_n}},$$

which has the standard comparative advantage interpretation. If Home has higher intermediate good price, then its input costs are higher and so it produces fewer varieties for itself. On the other hand, if the planner has higher value for Foreign's intermediate good, then the input costs for imports increase, which implies Home expands its import threshold and produces more for itself.

4.3.2 Goods for Foreign consumers

Now consider goods for Foreign consumers. As shown in Section 2.5, Foreign can supply for itself at cost $a_j^* g(p_e, p_N^*)$. So, its marginal utility of consumption is bounded above by:

$$u^{*\prime}(C_g^*)(Z_g^*)^{1/\sigma^*}(y_j^* + x_j)^{-1/\sigma^*} \le a_j^* g(p_e, p_N^*).$$

First order condition with respect to x_j is:

$$-\tau a_j g(\lambda_e, \lambda_N) + \lambda_N^* (Z_g^*)^{1/\sigma^*}(y_j^* + x_j)^{-1/\sigma^*} \le 0$$

$$\implies u^{*\prime}(C_g^*)(Z_g^*)^{1/\sigma^*}(y_j^* + x_j)^{-1/\sigma^*} \le \frac{u^{*\prime}(C_g^*)}{\lambda_N^*}\tau a_j g(\lambda_e, \lambda_N),$$

with equality if $x_j > 0$. Hence, the threshold good that equates the marginal utility of consumption in Foreign is characterized by

$$A(j_0) = \frac{p_N^*}{\lambda_N^*}\frac{\tau g(\lambda_e, \lambda_N)}{g(p_e, p_N^*)}.$$

For goods $j < j_0$, Home supplies them to Foreign at price $(p_N^*/\lambda_N^*)\tau a_j g(\lambda_e, \lambda_N)$ while its cost of inputs is $\tau a_j g(\lambda_e, \lambda_N)$.

The good that separates Home's export comparative advantage is characterized by

$$A(j_{0,c}) = \frac{\tau g(\lambda_e, \lambda_N)}{g(p_e, p_N^*)},$$

which implies that $j_0 > j_{0,c}$[8].

Home subsidizes goods to affect both the intensive and extensive margins. This arises because Home values the composite good in Foreign at price λ_N^*, higher than its costs p_N^*. Since the planner cannot specify the energy and intermediate good intensities for Foreign production for itself, it attempts to change the composition of the composite good. By subsidizing exports, more cleanly produced goods from Home are consumed and implicitly used in Foreign.

The intensive margin subsidy crucially depends on the elasticity of substitution across goods in Foreign, σ^*. Recall that:

$$p_N^* = \left(\int_0^1 (p_j^*)^{1-\sigma^*} dj \right)^{1/(1-\sigma^*)},$$

is the price of the intermediate good in Foreign. If goods are not substitutable in Foreign (small σ^*), then p_N^* is higher and approaches λ_N^*. On the other hand, if σ^* were large, then substitution across goods is high which drives down p_N^*. Intuitively, Home subsidizes goods for which it has comparative advantage so that consumers in Foreign substitute away from Foreign production for itself.

4.4 Exports further subsidized by Home

We have seen that Home subsidizes exports because it has a higher valuation for the composite good in Foreign. By pricing exports below cost, Home is able to change the composition of the composite good in Foreign. There are potentially additional gains to the planner because Foreign production for itself employs energy and intermediate good intensities that are not optimal from the perspective of the planner. In this subsection, I try to characterize the extent to which Home subsidizes on the extensive margin, in addition to the previous subsidies.

4.4.1 Crowding out consumption

I show in Appendix B.1 that consumption is fixed according to Table 3. Hence I substitute, noting that Home can crowd out Foreign production for itself (y_j^*) with exports for goods

[8] I show later that $\lambda_N^* > p_N^*$ for $\varphi^W > 0$.

Table 3: Consumption quantities in Home and Foreign

Home	$y_{cj} = \eta\left(a_j g(\lambda_e, \lambda_N)\right)^{-\sigma}$	$j < \bar{j}_m,$	$m_{cj} = \eta\left(\tau^* a_j^* g(\lambda_e, p_N^*)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_{cj} = \eta^*\left(\frac{p_N^*}{\lambda_N^*}\tau a_j g(\lambda_e, \lambda_N)\right)^{-\sigma^*}$	$j \leq j_0,$	$c_j^* = \eta^*\left(a_j^* g(p_e, p_N^*)\right)^{-\sigma^*}$	$j > j_0$

y_{cj} is the quantity consumed that is produced by Home for itself, similarly for x_{cj} and m_{cj}. c_j^* is the quantity that Foreign consumes at the price for which it can produce for itself (p_j^{y*}), but I do not make a statement on where it is produced.

$j > j_0$,

$$y_j = \frac{Z_g}{C_g} y_{cj}, j < \bar{j}_m; \quad m_j = \frac{Z_g}{C_g} m_{cj}, j > \bar{j}_m$$

$$x_j = \frac{Z_g^*}{C_g^*} x_{cj}, j < j_0; \quad y_j^* = \frac{Z_g^*}{C_g^*} c_j^* - x_j, j > j_0.$$

Dropping terms that do not depend on x_j for $j > j_0$ and noting that the planner's value of production is equal to its value of consumption (which cancels the Home goods market constraint), the Lagrangian becomes:

$$\mathcal{L} = -\frac{Z_g^*}{C_g^*}\left(\int_0^{j_0} \tau a_j g(\lambda_e, \lambda_N) x_{cj} dj + \int_{j_0}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) c_j^* dj - \lambda_N^* C_g^*\right)$$
$$- \int_{j_0}^1 \tau a_j g(\lambda_e, \lambda_N) x_j dj + \int_{j_0}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) x_j dj.$$

4.4.2 Optimality condition

Differentiating the Lagrangian with respect to x_j gives

$$\frac{\partial \mathcal{L}}{\partial x_j} = -\lambda_N^* \frac{\partial Z_g^*}{\partial x_j} V_s - \tau a_j g(\lambda_e, \lambda_N) + a_j^* g(p_e, p_N^*) + (\lambda_e - p_e) e_j^{y*} + (\lambda_N^* - p_N^*) n_j^{y*}, \tag{3}$$

where

$$V_s = \frac{\int_0^{j_0} \tau a_j g(\lambda_e, \lambda_N) x_{cj} dj + \int_{j_0}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) c_j^* dj - \lambda_N^* C_g^*}{\lambda_N^* C_g^*},$$

is the difference between planner's value of Foreign production and consumption as a share of value of consumption. $\bar{j}_x > j_0$ exists if the partial derivative is positive for $j \in [j_0, \bar{j}_x)$ and negative for $j > \bar{j}_x$. The partial derivative is positive if

$$(\lambda_e - p_e) e_j^{y*} + (\lambda_N^* - p_N^*) n_j^{y*},$$

which are the planner's values of energy and intermediate goods saved in Foreign, are sufficiently large. If the derivative is negative for all $j > j_0$, then $\bar{j}_x = j_0$ is a corner solution. This differs from KW, where $\bar{j}_x > j_0$ always holds for $\varphi^W > 0$, because the planner is already subsidizing to expand the export threshold in j_0. In this model, $j_0 > j_{0,c}$ always holds for $\varphi^W > 0$. The values of energy and intermediate good saved may have been already been accounted for in expanding j_0 when the planner subsidized on the intensive margin.

The change in Foreign's total amount of the composite good available as Home exports change is:

$$\frac{\partial Z_g^*}{\partial x_j} = \frac{C_g^* N_{g,c}^m}{C} \tau n_j^x - \frac{C_g^*(C_g - N_{g,c}^y)}{C} n_j^{y*}, \quad j > j_0, \tag{4}$$

where

$$\frac{C_g^* N_{g,c}^m}{C}, \quad \frac{C_g^*(C_g - N_{g,c}^y)}{C} \geq 1,$$

are the intermediate good multipliers. They measure the gross amount of good that flows from Home to Foreign and the gross amount of good that flows from Foreign to Foreign per unit of net consumption. τn_j^x and n_j^{y*} measure the amount of intermediate good needed per unit of j used or consumed in Foreign. Together,

$$\frac{C_g^* N_{g,c}^m}{C} \tau n_j^x,$$

represents the gross amount of intermediate goods that is implicitly consumed by Foreign for each additional unit of x_j. Similarly,

$$-\frac{C_g^*(C_g - N_{g,c}^y)}{C} n_j^{y*},$$

is the gross amount of composite good that is saved from an additional unit of exports from Home.

The gross amount of the composite good in Foreign, Z_g^*, may increase or decrease with an additional unit of x_j. If Home is inefficient at using the intermediate good in production (high τn_j^x) or if more gross output is required per unit of net output (high $\frac{C_g^* N_{g,c}^m}{C}$), then $\frac{\partial Z_g^*}{\partial x_j}$ is positive: increasing exports increases the total amount of goods in Foreign. Similarly, if Foreign is inefficient at using the intermediate good or has a high gross intermediate good multiplier then $\frac{\partial Z_g^*}{\partial x_j}$ would be negative. In general, the intermediate good multiplier is higher for domestic good flows (from Home to Home or Foreign to Foreign), due to home bias. Thus, it is likely that $\frac{\partial Z_g^*}{\partial x_j}$ is negative. If $\frac{\partial Z_g^*}{\partial x_j} > 0$ then it defeats the purpose of crowding out Foreign production, because increasing exports increases total amount of good embodied in Foreign production and consumption.

4.4.3 Characterizing $\bar{j}_x$

Supposing that $\bar{j}_x > j_0$ exists, I give a characterization of how much the planner wishes to expand the export threshold. Substituting equation (4) into equation (3) and setting it to 0, the threshold good $\bar{j}_x$ is characterized by:

$$A(\bar{j}_x) = \frac{\tau g(\lambda_e, \lambda_N) + \lambda_N^* \tau g_2(\lambda_e, \lambda_N) \frac{N_{g,c}^m C_g^*}{C} V_s}{g(p_e, p_N^*) + (\lambda_e - p_e) g_1(p_e, p_N^*) + g_2(p_e, p_N^*)((\lambda_N^* - p_N^*) + \lambda_N^* \frac{(C_g - N_{g,c}^y) C_g^*}{C} V_s)}.$$

For goods $j < \bar{j}_x$, Home exports them to Foreign, while Foreign produces for itself goods $j > \bar{j}_x$.

In the numerator,

$$\lambda_N^* \tau g_2(\lambda_e, \lambda_N) \frac{C_g^* N_{g,c}^m}{C} V_s,$$

increases $A(\bar{j}_x)$, which reduces $\bar{j}_x$. When Home increases exports, there are composite goods embodied in it which flow to Foreign. The planner wants to limit the amount of good that flows from Home to Foreign. Otherwise, it defeats the purpose of crowding out Foreign production to save resources.

In the denominator, there are three terms that increase the export threshold, the first two are direct effects while the third one is indirect. $(\lambda_e - p_e) g_1(p_e, p_N^*)$ and $(\lambda_N^* - p_N^*) g_2(p_e, p_N^*)$ capture planner's value difference of energy and intermediate goods saved from directly crowding out Foreign production for itself. The third term

$$\lambda_N^* \frac{C_g^* (C_g - N_{g,c}^y)}{C} V_s g_2(p_e, p_N^*),$$

captures the indirect effect of crowding out Foreign production. It represents the gross amount of composite good saved in Foreign. Reducing net production in Foreign implies a reduction in gross production, in addition to saving resources used directly to manufacture the good.

4.5 Shadow price the intermediate good and value of production and consumption

The planner subsidizes exports on the extensive margin because it wants to crowd out Foreign production for itself. Home can similarly subsidize production for itself if it cannot control how Foreign manufactures imports for Home. To find the planner's shadow value of the composite good in Foreign, I solve the model as if the planner wants to subsidize Home production for itself. Employing a similar strategy to subsidizing exports as done in Section

4.4, I show in Appendix B.2 that it is characterized by:

$$\int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_N) x_{cj} dj + \int_{\bar{j}_x}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) y_{cj}^* dj - \lambda_N^* C_g^* = 0. \tag{5}$$

This expression states that planner's value of Foreign consumption is equal to its value of Foreign production. The planner's value of Foreign production is higher than the actual cost because the planner values energy at a higher price than the world energy price. This expression implicitly characterizes λ_N^* while also having an intuitive interpretation.

I substitute equation 5 into V_s to get:

$$V_s = \frac{\int_{j_0}^{\bar{j}_x} (\lambda_e - p_e) e_j^{y*} x_{cj} + (\lambda_N^* - p_N^*) n_j^{y*} x_{cj} - (\tau a_j g(\lambda_e, \lambda_N) - a_j^* g(p_e, p_N^*)) x_{cj} dj}{\lambda_N^* C_g^*},$$

which is proportional between the planner's value gained from subsidizing exports and the value of the subsidies. If $V_s > 0$, then the planner has positive net gains from subsidizing exports on the extensive margin.

Planner's value for Home composite good is already solved and is characterized by:

$$\lambda_N = u'(C_g).$$

The value of consumption is equal to value of production:

$$\lambda_N C_g = \int_0^{\bar{j}_m} a_j g(\lambda_e, \lambda_N) dj + \int_{\bar{j}_m}^1 \tau^* a_j^* g(\lambda_e, \lambda_N^*) dj. \tag{6}$$

4.6 Outer problem

Now consider the outer problem. The planner chooses p_e and Q_e to maximize utility. The supply side of the problem is unchanged from KW, so I do not repeat the optimality condition derivations. The planner optimizes over p_e because it can control either net exports of energy or world energy prices. I chose p_e so that the planner does not control net exports of energy. I characterize differences between planner's value of energy and the world energy price as well as between planner's value of Foreign's composite good and its actual price.

The Lagrangian is (after substituting consumption values):

$$
\begin{aligned}
\mathcal{L} = {} & u(C_g) + u^*(C_g^*) - \varphi^W Q_e^W - L_e^W + \lambda_e Q_e^W - \lambda_N C_g - \lambda_N^* C_g^* \\
& - \frac{Z_g}{C_g}\left(\int_0^{\bar{j}_m} a_j g(\lambda_e, \lambda_N) y_{cj} dj + \int_{\bar{j}_m}^1 \tau^* a_j^* g(\lambda_e, \lambda_N^*) m_{cj} dj - \lambda_N C_g\right) \\
& - \frac{Z_g^*}{C_g^*}\left(\int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_N) x_{cj} dj + \int_{\bar{j}_x}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_N^* n_j^{y*}) c_j^* dj - \lambda_N^* C_g^*\right),
\end{aligned}
$$

where from the characterizations of λ_N and λ_N^* (equations 5 and 6), the second and third lines cancel.

4.6.1 Goods prices

Using the expressions for $\lambda_N^* C_g^*$ and $p_N^* C_g^*$, I solve for planner's value of Foreign's composite good, in terms of Foreign's goods prices:

$$
\lambda_N^* = p_N^* \frac{L_{g,c}^{y*} + \lambda_e C_{e,c}^{y*} + \int_{j0}^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_N) x_{cj} dj}{L_{g,c}^{y*} + p_e C_{e,c}^{y*} + \int_{j0}^{\bar{j}_x} a_j^* g(p_e, p_N^*) x_{cj} dj}.
$$

It follows that if $\lambda_e \geq p_e$, then $\lambda_N^* \geq p_N^*$. The planner always values the composite good in Foreign more than its cost due to the planner's higher valuation for energy. The planner's value of the composite good in Foreign is proportional to the price index of the intermediate good (p_N^*) and the ratio of the planner's value to the actual cost of Foreign's labour and energy use in producing for itself.

The planner's value difference for the composite good in Foreign is:

$$
\lambda_N^* - p_N^* = \frac{(\lambda_e - p_e) C_{e,c}^{y*} + \int_{j0}^{\bar{j}_x} (\tau a_j g(\lambda_e, \lambda_N) - a_j^* g(p_e, p_N^*)) x_{cj} dj}{L_{g,c}^{y*} + p_e C_{e,c}^{y*} + \int_{j0}^{\bar{j}_x} a_j^* g(p_e, p_N^*) x_{cj} dj} p_N^*.
$$

This can be computed directly because Home knows perfectly the composition of Foreign's intermediate good. This difference only depends on Foreign's energy consumption for production for itself ($C_{e,c}^{y*}$). It does not depend on the energy consumption values for other origin-destination pairs because the planner can regulate those directly (since the planner regulates production intensities for everything consumed and produced at Home).

4.6.2 Energy price condition

Now I optimize over the energy price, p_e. By the envelope condition, $\partial\mathcal{L}/\partial C_g = \partial\mathcal{L}/\partial\lambda_N = \partial\mathcal{L}/\partial\lambda_N^* = 0$. The first order condition with respect to the energy price is:

$$\frac{\partial\mathcal{L}}{\partial p_e} = \frac{\partial u^*(C_g^*)}{\partial p_e} - \varphi^W\frac{\partial Q_e^*}{\partial p_e} - p_e\frac{\partial Q_e^*}{\partial p_e} + \lambda_e\frac{\partial Q_e^*}{\partial p_e} - \lambda_N^*\frac{\partial C_g^*}{\partial p_e}.$$

Appendix B.3 simplifies the expression to yield an intuitive characterization of the border adjustment on energy:

$$\begin{aligned}(\lambda_e - \varphi^W - p_e)\frac{\partial Q_e^*}{\partial p_e} = &\int_0^{j_0}\left(1-\frac{p_N^*}{\lambda_N^*}\right)\tau a_j g(\lambda_e,\lambda_N)\frac{\partial x_{cj}}{\partial p_e}dj + \int_{j_0}^{\bar{j}_x}(\tau a_j g(\lambda_e,\lambda_N) - a_j^* g(p_e,p_N^*))\frac{\partial c_j^*}{\partial p_e}\\ &+ (\lambda_e - p_e)\frac{\partial C_{e,c}^{y*}}{\partial p_e} + (\lambda_N^* - p_N^*)\frac{\partial N_{g,c}^{y*}}{\partial p_e}.\end{aligned}$$

In this case, beyond the three wedges in KW[9], the planner additionally considers the wedge between planner's marginal valuation of the intermediate good used in Foreign and its price there (second term on the second line). The fact that this wedge emerges is not surprising. Similar to $C_{e,c}^{y*}$, which is not under the planner's control, $N_{g,c}^{y*}$, representing intermediate good use for Foreign supplying final products for itself, is also not controlled by the planner. I rearrange the terms to solve for the difference between planner's value for energy and the energy price:

$$\lambda_e - p_e = \frac{\varphi^W\partial Q_e^*/\partial p_e - \sigma^*\left(\frac{\partial p_N^*/\partial p_e}{p_N^*}S_1 + \frac{\partial g(p_e,p_N^*)/\partial p_e}{g(p_e,p_N^*)}S_2\right) + (\lambda_N^* - p_N^*)\partial N_{g,c}^{y*}/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_{e,c}^{y*}/\partial p_e},$$

where

$$S_1 = \int_0^{j_0}\left(1-\frac{p_N^*}{\lambda_N^*}\right)\tau a_j g(\lambda_e,\lambda_N)x_{cj}dj, \quad S_2 = \int_{j_0}^{\bar{j}_x}(\tau a_j g(\lambda_e,\lambda_N) - a_j^* g(p_e,p_N^*))x_{cj}dj,$$

are the export subsidies and

$$N_{g,c}^{y*} = \int_{\bar{j}_x}^1 n_j^{y*}c_j^* dj, \quad C_{e,c}^{y*} = \int_{\bar{j}_x}^1 e_j^{y*}c_j^* dj,$$

are the intermediate good and energy consumption for the Foreign production for final consumed product.

[9] The left hand side is the extraction wedge, which is the wedge between planner's marginal valuation of energy extracted and energy price. The first two terms on the right hand side are export wedges. They are wedges between Home's shadow cost of supplying exports and Foreign's marginal utility from consuming them. The third term represents the consumption wedge. It is the wedge between planner's marginal valuation of energy used and energy price.

5 Implementation

There are several equivalent ways to implement the optimal policy, I describe one implementation that uses an extraction tax and border adjustments on imports and exports.

Let

$$t_b^e = \lambda_e - p_e = \frac{\varphi^W \partial Q_e^*/\partial p_e - \sigma^* \left(\frac{\partial p_N^*/\partial p_e}{p_N^*} S_1 + \frac{\partial g(p_e,p_N^*)/\partial p_e}{g(p_e,p_N^*)} S_2 \right) + (\lambda_N^* - p_N^*) \partial N_{g,c}^{y*}/\partial p_e}{\partial Q_e^*/\partial p_e - \partial C_{e,c}^{y*}/\partial p_e},$$

be the border adjustment on energy. It is inversely proportional to the amount of export subsidies; the planner accounts for the magnitude of the subsidies while considering the border adjustment on energy. If the planner values energy at a high price, then it will subsidize exports more heavily, which incurs a higher cost to Home. It is also related to the planner's value difference for Foreign's composite intermediate good, $\lambda_N^* - p_N^*$.

Furthermore, let

$$t_b^n = \lambda_N^* - p_N^* = \frac{S_2 + (\lambda_e - p_e) C_{e,c}^{y*}}{L_{g,c}^{y*} + p_e C_{e,c}^{y*} + \int_{j_0}^{\bar{j}_x} a_j^* g(p_e, p_N^*) c_j^* dj} p_N^*,$$

be the border adjustment on the intermediate good used in production. It depends on the planner's value difference of energy in Foreign, $\lambda_e - p_e$. If $\lambda_e - p_e$ is high, then the planner is more unsatisfied with Foreign's energy consumption, so it increases the border adjustment on the intermediate good to reduce embodied energy in imports.

The implementation is based on the one proposed in KW. First, Home imposes a nominal extraction tax equal to the global social cost of carbon, $t_e^N = \varphi^W$. Then, it adds a border adjustment of t_b^e on all energy traded. Energy imports are taxed at rate t_b^e while energy exports receive a subsidy at rate t_b^e. Thus, the energy price faced by users at Home is $p_e + t_b^e$.

For traded goods, imports are taxed at rate t_b^e for direct emissions associated with their production (for example emissions used to heat input material). They would additionally be taxed at rate t_b^n for each unit of the intermediate good used during production, to account for embedded emissions. Hence, the price faced by users of imports is

$$\tau^* l_j^m + \tau^* p_e e_j^m + \tau^* p_N^* n_j^m + t_b^e \tau^* e_j^m + t_b^n \tau^* n_j^m = \tau^* a_j^m g(\lambda_e, \lambda_N^*).$$

For exports, producers receive a subsidy, not compensate them for the more stringent carbon policy at Home (hence not an adjustment on energy), but to shift the composition of Foreign's composite good. This would make Foreign's composite good consist of fewer energy-intensive goods and hence cleaner. The subsidy is at rate:

$$\left(1 - \frac{p_N^*}{\lambda_N^*}\right) \tau a_j g(\lambda_e, \lambda_N),$$

per unit of exported good for goods $j < j_0$, and at rate

$$\tau a_j g(\lambda_e, \lambda_N) - a_j^* g(p_e, p_N^*),$$

for goods $j_0 \leq j \leq \bar{j}_x$ (noting the possibility that $\bar{j}_x = j_0$). Consequently, the prices faced by Foreign are

$$p_j^x = \begin{cases} \frac{p_N^*}{\lambda_N^*} \tau a_j g(\lambda_e, \lambda_N) & j < j_0 \\ a_j^* g(p_e, p_N^*) & j_0 \leq j \leq \bar{j}_x \end{cases}$$

Finally, I show that the border adjustment is not on the full energy content of the intermediate good. The energy embodied in a unit of Foreign's composite good is:

$$\alpha_e^* = \frac{N_{g,c}^x (C_{e,c}^y + C_{e,c}^m)}{M} + \frac{(C_g - N_{g,c}^y)(C_{e,c}^{y*} + C_{e,c}^x)}{M},$$

which captures the gross flow of energy from Home to Foreign and from Foreign to itself. The border adjustment, if applied to the full energy content, is $t_b^e \alpha_e^*$. But the adjustment on the intermediate good, t_b^n, depends only on $C_{e,c}^{y*}$, which is Foreign's energy consumption in production for itself, as opposed to the other energy consumption values which are subject to Home's regulations. Thus, $t_b^n \neq t_b^e \alpha_e^*$ suggests that Home adjusts its import policy to account for production that occurred in a region with carbon policies.

6 Relationship to CBAM

I use the model to analyze the EU's Carbon Border Adjustment Mechanism (CBAM). Let Home be the EU, and Foreign be the rest of the world, and suppose that only Home implements any carbon policy. I begin with a description of the implementation, illustrate the different types of emissions, and discuss similarities and differences between the optimal policy and CBAM on traded goods.

As discussed more analytically in Section 5, the optimal policy can be implemented with a combination of an extraction tax and border adjustments. Specifically, energy exporters receive a rebate while energy importers are taxed. For traded goods, exporters receive a subsidy per unit of good exported. Goods importers, on the other hand, pay taxes based on the embodied emissions of the imported good. The optimal policy proposes different values of the export subsidy based on comparative advantage. For goods with strong Home comparative advantage, the planner prices them at a fraction of the cost. The fraction is determined by the ratio of the price and the planner's value of the intermediate good abroad. Hence, the higher the planner's value, the more aggressive the subsidies. For goods with weak Home comparative advantage, the subsidy serves to make up the gap between Foreign's price and Home's cost.

I use an example to illustrate the various types of emissions, as classified by the EU. There are three notions of carbon emission from the production of goods: direct, indirect and embedded[10]. Direct emissions are a result of production, such as combusting fossil fuel in the production process to heat/cool input material, or emissions directly emitted by the input material. Indirect emissions are emissions embodied in electricity used. Embedded emissions are emissions embodied in intermediate (precursor) goods. As an example, to produce cement, clinker is a necessary precursor good. Clinker is produced from mixing limestone with other raw material, and heating them to around 1450 degrees Celsius. In the process of converting limestone to clinker, the chemical reaction causes the release of CO2. Hence, in the production of clinker, emissions associated with combusting fossil fuel to heat inputs and emissions from limestone's chemical reaction are direct emissions. Any electricity used in production contributes to indirect emissions. After clinker is produced, it is used in the production of cement. As a result, direct and indirect emissions from clinker production become embedded emissions in cement production.

Now I compare the optimal policy to CBAM. The optimal import border adjustment resembles CBAM. While the optimal export subsidies are politically infeasible to implement, they reveal incentives for countries intending to unilaterally implement carbon policies and offer insights into revising trade regulations to tackle climate issues.

In CBAM, imports into the EU are taxed on both energy used in production (direct and indirect emissions), and on energy embodied in intermediate goods (embedded emissions). Rebates are offered for intermediates produced in countries with carbon regulations. Goods importers are required to declare all embodied emissions. While direct and indirect emissions are straightforward to calculate, embedded emissions calculations are more burdensome. According to the guidance document for CBAM, importers need to report emissions for all precursor goods. If the precursor good itself is a complex good[11], then the process is "repeated recursively until no more precursors are relevant". This ensures that all embodied emissions are reported and regulated in CBAM.

The primary goal of CBAM is to ensure that producers of imported goods adhere to the same carbon regulations as those in the EU. The model suggests along the same lines: (i) producers in Foreign should face the planner's valuation of energy and the intermediate good, as opposed to market prices and (ii) the border adjustment on embodied emissions should only apply to the share of the intermediate good not produced under any carbon policies[12]. As shown in Section 5, similar to the import rebates in CBAM, the planner only considers the share of the intermediate good that is produced in Foreign using inefficient

[10]They correspond to scope 1, 2 and 3 emissions, as used in ESG reportings.

[11]CBAM classifies two types of goods: simple and complex. Simple goods combine raw material with energy, while complex goods additionally uses simple goods (as precursors).

[12]In this case, Home does not impose that import producers face the same intermediate good price at Home, due to differential trade costs.

production intensities when applying a tax on embodied emissions.

While CBAM does not implement any export policies due to political and WTO concerns, the optimal policy proposes export subsidies. The subsidies are not meant to compensate manufacturers for the more stringent carbon regulation at Home, rather they are aimed at reducing emissions in Foreign. Contrary to KW, the optimal subsidies expand both the intensive and extensive margins: Home produces more of every good it exports and exports a wider range of goods. Home expands the intensive export margin to make the composite good in Foreign cleaner and expands the extensive margin to directly crowd out Foreign's production for itself.

Due to international trade, Home subsidizes exports on the intensive margin to reduce embodied emissions in Home imports. This novel insight is a direct consequence of including intermediate goods. In KW, all goods are consumed whereas in my model, a portion of Home exports is used in Foreign production which are then implicitly imported back by Home. For each good that Home imports, it embodies less carbon if the intermediate good were cleaner. Therefore, subsidizing on the intensive export margin that changes the composition of the intermediate good in Foreign reduces embodied emissions in Home imports. The extent of the subsidy depends on Foreign's elasticity of substitution across goods. If the elasticity is high, then Home is incentivized to increase production of goods for which it has strong comparative advantage. As a result, consumers in Foreign substitute towards the cheaper exports from Home, which reduces dirty production in Foreign. Conversely, if substitution across goods is relatively low, Home reduces its subsidy on the intensive margin.

Subsidies on the extensive margin will always be present. While the intensive margin expansion shifts demand and indirectly reduces Foreign production, the extensive margin expansion crowds out Foreign production directly. For goods that Foreign produces for itself, the production intensities are less efficient than the planner's optimal intensities. For each good, the planner saves energy and the intermediate good if it were produced at Home instead. Therefore, Home subsidizes and exports goods above the comparative advantage threshold that are not too costly for it to produce.

I differentiate the extensive and intensive margin export subsidies and capture the effect of altering the composition of the intermediate good with an example. Suppose that both Home and Foreign have sizeable steel manufacturing industries that produce different but highly substitutable steel products. The optimal policy dictates that Home subsidizes current steel exports (intensive margin subsidy) to increase demand for Home's steel in Foreign. This shifts the composition of cars manufactured in Foreign and reduces the embodied emissions in them because a greater share of the steel originates from Home. In addition, when Home imports these cars, the border adjustment on embodied emissions does not apply to the portion of the steel that came from Home. Note that this differs from the export subsidy aimed at expanding the extensive margin; Home is already exporting steel products before the subsidy.

The model illustrates the importance of subsidizing exports, suggesting that trade regulations should make exceptions for subsidies that protect the environment. The argument against export subsidies is that they introduce unfair competition which harms manufacturers. However, climate change is a global externality and countries with weak environmental regulations harm people worldwide.

7 Conclusion

This paper extends Kortum and Weisbach (2023) by introducing intermediate goods. With intermediates, the model can be more seamlessly calibrated to carbon flows for quantitative simulation. A deeper analytical insight is that the optimal policy in the extended model more closely relates to, and largely supports, the EU's latest carbon policy, CBAM. In future work, I intend to explore how this model fits in the framework of Eaton and Kortum (2002) and Caliendo and Parro (2015), with multiple countries and multiple industries, which would allow substitution across intermediate goods. I can then evaluate the extent to which CBAM's tax on embedded emissions gets manufactures to choose more environmentally friendly intermediates.

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A Competitive equilibrium

Assume function forms

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}, \quad a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta}, \quad A(j) = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta}$$

and characterize the import/export thresholds as

$$A(\bar{j}_m) = \frac{g(p_e, p_n)}{\tau^* g(p_e, p_n^*)}, \quad A(\bar{j}_x) = \frac{\tau g(p_e, p_n)}{g(p_e, p_n^*)}$$

A.1 Share of energy to import/export threshold

We know that consumption is fixed by minimum cost of production. Then it follows that

$$\begin{aligned}
C_{e,c}^y &= \int_0^{\bar{j}_m} e_j^y y_{cj} dj \\
&= \eta g_1(p_e, p_n) g(p_e, p_n)^{-\sigma} \int_0^{\bar{j}_m} a_j^{1-\sigma} dj \\
&= \eta \tilde{g}'(p_e) \tilde{g}(p_e)^{-\sigma} p_n^{\alpha_n} p_n^{-\alpha_n \sigma} (\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta} \\
C_{e,c}^m &= \eta (\tau^*)^{-\sigma} g_1(p_e, p_n^*) g(p_e, p_n^*)^{-\sigma} \int_{\bar{j}_m}^1 (a_j^*)^{1-\sigma} dj \\
&= \eta \tilde{g}'(p_e) \tilde{g}(p_e)^{-\sigma} (\tau^*)^{1-\sigma} (p_n^*)^{\alpha_n} (p_n^*)^{-\alpha_n \sigma} (1-\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{(A^*)^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta}
\end{aligned}$$

$$\begin{aligned}
\frac{C_{e,c}^y}{C_{e,c}^y + C_{e,c}^m} &= \frac{p_n^{\alpha_n(1-\sigma)} \bar{j}_m^{1+(1-\sigma)/\theta} A^{-(1-\sigma)/\theta}}{p_n^{\alpha_n(1-\sigma)} \bar{j}_m^{1+(1-\sigma)/\theta} A^{-(1-\sigma)/\theta} + (\tau^*)^{1-\sigma} (p_n^*)^{\alpha_n(1-\sigma)} (1-\bar{j}_m)^{1+(1-\sigma)/\theta} (A^*)^{-(1-\sigma)/\theta}} \\
&= \frac{A^{-(1-\sigma)/\theta}}{A^{-(1-\sigma)/\theta} + (\tau^*)^{1-\sigma} (p_n^*/p_n)^{\alpha_n(1-\sigma)} ((1-\bar{j}_m)/\bar{j}_m)^{1+(1-\sigma)/\theta} (A^*)^{-(1-\sigma)/\theta}} \\
&= \frac{A^{-(1-\sigma)/\theta}}{A^{-(1-\sigma)/\theta} + ((\tau^*)^{-\theta} (p_n/p_n^*)^{\alpha_n \theta} \frac{\bar{j}_m}{(1-\bar{j}_m)} A^*)^{-(1-\sigma)/\theta} \frac{(1-\bar{j}_m)}{\bar{j}_m}} \\
&= \frac{A^{-(1-\sigma)/\theta}}{A^{-(1-\sigma)/\theta} + A^{-(1-\sigma)/\theta} \frac{1-\bar{j}_m}{\bar{j}_m}} \\
&= \bar{j}_m
\end{aligned}$$

which is what I desired to show. I employ a similar argument to show that

$$\frac{C^x_{e,c}}{C^{y*}_{e,c} + C^x_{e,c}} = \bar{j}_x$$

A.2 Expressing energy embodied in final demand in terms of energy used for consumption

Table 1 shows that energy embodied in final demand is a linear combination of energy used to produce for consumption at Home and Foreign. With the same functional form assumptions, we can rewrite it so that energy embodied in final demand is only a function of energy used for consumption.

$$\begin{aligned}
C^y_{e,t} &= M\left(C^y_{e,c} + \frac{\alpha_n \bar{j}_x}{1-\alpha_n(1-\bar{j}_x)} C^m_{e,c}\right) \\
&= D(1-\alpha_n(1-\bar{j}_x)C^y_{e,c} + \alpha_n \bar{j}_x C^m_{e,c}) \\
&= D\eta \tilde{g}'(p_e)\tilde{g}(p_e)^{-\sigma} p_n^{\alpha_n(1-\sigma)} (\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta} \\
&\quad \left((1-\alpha_n(1-\bar{j}_x)) + \alpha_n \bar{j}_x \left(\frac{p_n^*}{p_n}\right)^{\alpha_n(1-\sigma)} \left(\frac{1-\bar{j}_m}{\bar{j}_m}\right)^{1+(1-\sigma)/\theta} (\tau^*)^{1-\sigma} \frac{(A^*)^{-(1-\sigma)/\theta}}{A^{-(1-\sigma)/\theta}}\right) \\
&= D\eta \tilde{g}'(p_e)\tilde{g}(p_e)^{-\sigma} p_n^{\alpha_n(1-\sigma)} (\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta} \\
&\quad \left((1-\alpha_n(1-\bar{j}_x) + \alpha_n \bar{j}_x \frac{1-\bar{j}_m}{\bar{j}_m} \left(\left(\frac{p_n}{p_n^*}\right)^{\alpha_n\theta} (\tau^*)^{-\theta} \frac{A^*}{A} \frac{A}{(\tau^*)^{-\theta}(p_n/p_n^*)^{\alpha\theta} A^*}\right)^{-(1-\sigma)/\theta}\right) \\
&= D\eta \tilde{g}'(p_e)\tilde{g}(p_e)^{-\sigma} p_n^{\alpha_n(1-\sigma)} (\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta} \left(1-\alpha_n(1-\bar{j}_x) + \alpha_n \bar{j}_x \frac{1-\bar{j}_m}{\bar{j}_m}\right) \\
&= \eta \tilde{g}'(p_e)\tilde{g}(p_e)^{-\sigma} (\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta} \left(D p_n^{\alpha_n(1-\sigma)}(1-\alpha_n(1-\bar{j}_x) + \alpha_n \bar{j}_x \frac{1-\bar{j}_m}{\bar{j}_m})\right) \\
&= D\left(1-\alpha_n(1-\bar{j}_x) + \alpha_n \bar{j}_x \frac{1-\bar{j}_m}{\bar{j}_m}\right) c^{y0}_e \\
&= D\left(1-\alpha_n(1-\frac{\bar{j}_x}{\bar{j}_m})\right) c^{y0}_e
\end{aligned}$$

where

$$D = \frac{M}{1-\alpha_n(1-\bar{j}_x)} = \frac{M^*}{1-\alpha_n \bar{j}_m}$$

B Derivations for optimal policy

B.1 Consumption values

First consider goods for Home consumers. The marginal utilities are fixed by

$$u^{*\prime}((C_g/Z_g)y_j) = a_j g(\lambda_e, \lambda_n), \quad j < \bar{j}_m$$

$$u^{*\prime}((C_g/Z_g)m_j) = \tau^* a_j^* g(\lambda_e, \lambda_n^*), \quad j > \bar{j}_m$$

which implies

$$y_j = \frac{\eta}{C_g/Z_g}(a_j g(\lambda_e, \lambda_n))^{-\sigma}, \quad m_j = \frac{\eta}{C_g/Z_g}(\tau^* a_j^* g(\lambda_e, \lambda_n^*))^{-\sigma}$$

Substituting back into the definition of Z_g, we have

$$\begin{aligned}
Z_g &= \left(\int_0^{\bar{j}_m} y_j^{(\sigma-1)/\sigma} dj + \int_{\bar{j}_m}^1 m_j^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)} \\
&= \left(\int_0^{\bar{j}_m} \left(\eta \frac{Z_g}{C_g}(a_j g(\lambda_e, \lambda_n))^{-\sigma} \right)^{(\sigma-1)/\sigma} dj + \int_{\bar{j}_m}^1 \left(\eta \frac{Z_g}{C_g}(\tau^* a_j^* g(\lambda_e, \lambda_n^*))^{-\sigma} \right)^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)} \\
C_g &= \left(\int_0^{\bar{j}_m} y_{cj}^{(\sigma-1)/\sigma} dj + \int_{\bar{j}_m}^1 m_{cj}^{(\sigma-1)/\sigma} dj \right)^{\sigma/(\sigma-1)}
\end{aligned}$$

where

$$y_{cj} = \eta(a_j g(\lambda_e, \lambda_n))^{-\sigma}, \quad m_{cj} = \eta(\tau^* a_j^* g(\lambda_e, \lambda_n^*))^{-\sigma}$$

as desired. Similarly,

$$u^{*\prime}((C_g^*/Z_g^*)x_j) = \frac{p_n^*}{\lambda_n^*}\tau a_j g(\lambda_e, \lambda_n), \quad j < j_0$$

$$u^{*\prime}((C_g^*/Z_g^*)(y_j^* + x_j)) = a_j^* g(p_e, p_n^*), \quad j > j_0$$

which implies

$$x_j = \frac{\eta^*}{C_g^*/Z_g^*}((p_n^*/\lambda_n^*)\tau a_j g(\lambda_e, \lambda_n))^{-\sigma^*}, \quad y_j^* + x_j = \frac{\eta^*}{C_g^*/Z_g^*}(a_j^* g(p_e, p_n^*))^{-\sigma^*}$$

substituting back into the definition of Z_g^* to get

$$Z_g^* = \left(\int_0^{j_0} x_j^{(\sigma^*-1)/\sigma^*} dj + \int_{j_0}^1 (c_j^*)^{(\sigma^*-1)/\sigma^*} dj\right)$$
$$C_g^* = \left(\int_0^{j_0} x_{cj}^{(\sigma^*-1)/\sigma^*} dj + \int_{j_0}^1 (c_j^*)^{(\sigma^*-1)/\sigma^*} dj\right)$$

where

$$x_j = \eta^*((p_n^*/\lambda_n^*)\tau a_j g(\lambda_e, \lambda_n))^{-\sigma^*}, \quad c_j^* = \eta^*(a_j^* g(p_e, p_n^*))^{-\sigma^*}$$

B.2 Planner's value of the composite good in Foreign

To find planner's shadow value of the composite good in Foreign, I solve the model as if the planner wants to subsidize Home production for itself. I have shown that consumption quantities are fixed as in table 3. Employing a similar strategy to subsidizing exports, I consider the Lagrangian

$$\begin{aligned}\mathcal{L} = {} & u(C_g) + u^*(C_g^*) - \varphi^W Q_e^W - L_e^W + \lambda_e Q_e^W \\ & - \frac{Z_g^*}{C_g^*}\left(\int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_n) x_{cj} dj + \int_{\bar{j}_x}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_n^* n_j^{y*}) y_{cj}^* dj - \lambda_n^* C_g^*\right) \\ & - \int_{\bar{j}_m}^1 a_j g(\lambda_e, \lambda_n) y_j dj + \int_{\bar{j}_m}^1 \tau^* a_j^* g(\lambda_e, \lambda_n^*) y_j dj - \lambda_n C_g - \lambda_n^* C_g^*\end{aligned}$$

First order condition with respect to y_j for $j \geq \bar{j}_m$ is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial y_j} = {} & -a_j g(\lambda_e, \lambda_n) + \tau^* a_j g(\lambda_e, \lambda_n^*) \\ & - \frac{\partial Z_g^*}{\partial y_j}\frac{1}{C_g^*}\left(\int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_n) x_{cj} dj + \int_{\bar{j}_x}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_n^* n_j^{y*}) y_{cj}^* dj - \lambda_n^* C_g^*\right)\end{aligned}$$

Since the planner dictates Foreign's energy and intermediate good intensities, it must be the case that λ_n^* solves

$$\frac{\partial \mathcal{L}}{\partial y_{\bar{j}_m}} = 0$$

So it suffices if

$$\int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_n) x_{cj} dj + \int_{\bar{j}_x}^1 (l_j^{y*} + \lambda_e e_j^{y*} + \lambda_n^* n_j^{y*}) y_{cj}^* dj - \lambda_n^* C_g^* = 0 \tag{7}$$

which gives the characterization.

B.3 Energy price derivation

I rewrite

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = \int_0^{j_0} \frac{p_n^*}{\lambda_n^*} \tau a_j g(\lambda_e, \lambda_n) \frac{\partial x_{cj}}{\partial p_e} dj + \int_{j_0}^1 a_j^* g(p_e, p_n^*) \frac{\partial c_j^*}{\partial p_e} dj$$

and the partial derivative of value of Foreign consumption is

$$\frac{\partial \lambda_n^* C_g^*}{\partial p_e} = \int_0^{\bar{j}_x} \tau a_j g(\lambda_e, \lambda_n) \frac{\partial x_{cj}}{\partial p_e} dj + \int_{\bar{j}_x}^1 a_j^* g(p_e, p_n^*) \frac{\partial c_j^*}{\partial p_e} dj + (\lambda_e - p_e) \frac{\partial C_{e,c}^{y*}}{\partial p_e} + (\lambda_n^* - p_n^*) N_{g,c}^{y*}$$

which becomes the intuitive form with four wedges

$$(\lambda_e - \varphi^W - p_e) \frac{\partial Q_e^*}{\partial p_e} = \int_0^{j_0} \left(1 - \frac{p_n^*}{\lambda_n^*}\right) \tau a_j g(\lambda_e, \lambda_n) \frac{\partial x_{cj}}{\partial p_e} dj + \int_{j_0}^{\bar{j}_x} (\tau a_j g(\lambda_e, \lambda_n) - a_j^* g(p_e, p_n^*)) \frac{\partial c_j^*}{\partial p_e}$$
$$+ (\lambda_e - p_e) \frac{\partial C_{e,c}^{y*}}{\partial p_e} + (\lambda_n^* - p_n^*) \frac{\partial N_{g,c}^{y*}}{\partial p_e}$$

as required.